

Cosmic acceleration as the solution to the cosmological constant problem*

Philip D. Mannheim

Department of Physics, University of Connecticut, Storrs, CT 06269
mannheim@uconnvm.uconn.edu

Abstract

In this paper we provide both a diagnosis and resolution of the cosmological constant problem, one in which a large (as opposed to a small) cosmological constant Λ can be made compatible with observation. We trace the origin of the cosmological constant problem to the assumption that the local gravitational Newton constant G (as measured in a Cavendish experiment) sets the scale for global cosmology. And then we show that once this assumption is relaxed, the very same cosmic acceleration which has served to make the cosmological constant problem so very severe can instead then serve to provide us with its potential resolution. In particular, we present an alternate cosmology, one based on conformal gravity, a theory whose effective cosmological G_{eff} not only differs from the Cavendish one by being altogether smaller than it, but, by even being explicitly negative, naturally leads to cosmological repulsion. We show in the conformal theory, that once given only that the sign of Λ is specifically the negative one associated with spontaneous scale symmetry breaking, then, that alone, no matter how big Λ might actually be in magnitude, is sufficient to not only make the actually measurable contribution $8\pi G_{eff}\Lambda/3cH^2(t_0)$ of Λ to current era cosmology naturally be of order one today, but to even do so in a way which is fully compatible with the recent high z supernovae cosmology data. Thus to solve the cosmological constant problem we do not need to change or quench the energy content of the universe, but rather we only need change its effect on cosmic evolution.

I. DIAGNOSIS OF THE COSMOLOGICAL CONSTANT PROBLEM

The recent discovery [1,2] of a cosmic acceleration has made the already extremely disturbing cosmological constant problem even more vexing than before. Specifically, a phenomenological fitting to the new high z supernovae Hubble plot data using the standard Einstein-Friedmann cosmological evolution equations

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$$\dot{R}^2(t) + kc^2 = \dot{R}^2(t)(\Omega_M(t) + \Omega_\Lambda(t)) \quad (1)$$

$$\Omega_M(t) + \Omega_\Lambda(t) + \Omega_k(t) = 1 \quad (2)$$

$$q(t) = (n/2 - 1)\Omega_M(t) - \Omega_\Lambda(t) \quad (3)$$

where $\Omega_M(t) = 8\pi G\rho_M(t)/3c^2H^2(t)$ is due to ordinary matter (viz. matter for which $\rho_M(t) = A/R^n(t)$ where $A > 0$ and $3 \leq n \leq 4$), where $\Omega_\Lambda(t) = 8\pi G\Lambda/3cH^2(t)$ is due to a cosmological constant Λ and where $\Omega_k(t) = -kc^2/\dot{R}^2(t)$ is due to the spatial 3-curvature k , has revealed that not only must the current era $\Omega_\Lambda(t_0)$ actually be non-zero today, it is even explicitly required to be of order one. Typically, the allowed parameter space compatible with the available data is found to be centered on the line $\Omega_\Lambda(t_0) = \Omega_M(t_0) + 1/2$ or so with (the presumed positive) $\Omega_M(t_0)$ being found to be limited to the range $(0, 1)$ and $\Omega_\Lambda(t_0)$ to the range $(1/2, 3/2)$ or so, with the current ($n = 3$) era deceleration parameter $q(t_0) = (n/2 - 1)\Omega_M(t_0) - \Omega_\Lambda(t_0)$ thus having to approximately lie within the $(-1/2, -1)$ interval.¹ Thus, not only do we find that the universe is currently accelerating, but additionally we see that with there being no allowed $\Omega_\Lambda(t_0) = 0$ solution at all (unless $\Omega_M(t_0)$ could somehow be allowed to go negative), the longstanding problem (see e.g. [4,5] for some recent reviews) of trying to find some way by which $\Omega_\Lambda(t_0)$ (and thus $q(t_0)$) could be quenched by many orders of magnitude from its quantum gravity Planck temperature expectation or its typical $c|\Lambda| = \sigma T_V^4$ ($T_V \simeq 10^{16}$ °K) particle physics expectation has now been replaced by the need to find a specific such mechanism which in practice (rather than just in principle) would explicitly put $\Omega_\Lambda(t_0)$ into this very narrow $(1/2, 3/2)$ box. Not only is it not currently known how it might be possible to actually do this, up to now no mechanism has been identified which might even fix the sign of the standard model $\Omega_\Lambda(t_0)$ let alone its magnitude.

Now while such quenching of Λ has yet to be achieved, it is important to note that even if it were to actually be achieved, the very use of such a quenched Λ in Eq. (1) then creates a further problem for the standard model, one related to the initial conditions of the then associated early universe. Specifically, with both the current era $\Omega_M(t_0)$ and a such quenched $\Omega_\Lambda(t_0)$ then being of the same order of magnitude today (the so-called cosmic coincidence), the very existence of an early universe initial big bang singularity (i.e. of an infinite or overwhelmingly large $\dot{R}(t=0)$ in Eq. (1)) would then oblige the initial values of these same two functions to have to obey $\Omega_M(t=0) + \Omega_\Lambda(t=0) = 1$ (no matter what the value of k), to then entail that the initial conditions for Eq. (1) would have to be picked to incredible accuracy in order for the universe to actually be able to evolve into one in which $\Omega_M(t_0)$ and $\Omega_\Lambda(t_0)$ could then be of comparable magnitude today. As such this problem

¹The spread around the line $\Omega_\Lambda(t_0) = \Omega_M(t_0) + 1/2$ is of order $\pm 1/2$, to thus allow solutions in which the current era $\Omega_M(t_0)$ is negligible, with $\Omega_\Lambda(t_0)$ having to then lie in the $(0, 1)$ interval and $q(t_0)$ in $(0, -1)$. (For explicit acceptable $\Omega_M(t_0) = 0$ fits in the 'empty universe' case where $\Omega_\Lambda(t_0)$ is also zero see [3]). While completely foreign to the standard model, as we shall see below, universes where $\rho_M(t_0)$ has little effect on current era cosmic evolution can nonetheless actually occur quite naturally in the alternate conformal gravity theory which we explore in this paper.

is actually a variant of the original cosmological flatness problem, with it not so much mattering that $c\Lambda$ and $\rho_M(t_0)$ would have to be of the same order of magnitude today, but rather only that Λ would actually be non-zero at all, since once Λ is non-zero, $\Omega_M(t_0)$ could not then be identically equal to one today (the value it would take at all times in a $\Lambda = 0$ inflationary cosmology [6]), with it then being extremely difficult to understand why $\Omega_M(t_0)$ had not already been redshifted to zero this late after the big bang. Indeed, since a current era $\Omega_\Lambda(t_0) = \Omega_M(t_0) + 1/2$ universe would continue to accelerate into the future, future observers would see a progressively declining $\Omega_M(t)$ and a progressively increasing $\Omega_\Lambda(t)$, with the ensuing, no longer anywhere near close values of $\Omega_M(t)$ and $\Omega_\Lambda(t)$ specifically being such as to still necessitate early universe fine tuning. While any non-zero value for Λ thus necessitates early universe fine tuning no matter what that value might actually be, a universe in which $\Omega_\Lambda(t_0)$ is just a little bit bigger than $\Omega_M(t_0)$ today has a further peculiar feature, namely that $\Omega_M(t)$ would actually be bigger than $\Omega_\Lambda(t)$ at all redshifts above $z = 1$ or so. Consequently, such a universe would be one which would have decelerated continually all the time since the big bang and which would have only started to accelerate in just our own epoch. Since the conformal gravity theory we present below does not possess any such switch over (its universe accelerates both below and above $z = 1$), this switch over feature of the standard theory will actually serve both as a diagnostic for it and as way to distinguish between it and the conformal theory, a point to which we shall return below. We thus recognize two separate aspects to the cosmological constant problem, namely the need to find a way to quench Λ down from its fundamental physics expectation in the first place, and the need to then find a way to fix initial conditions in the early universe so as to be able to accommodate the non-zero quenched value for Λ (and the thus non-unity value for $\Omega_M(t_0)$) which would then ensue.

Now while a solution to the cosmological constant problem might yet be found within standard gravity, the above described situation is so disquieting (with the cosmological constant problem having resisted solution for such a very long time now) as to suggest that in fact there might actually be something basically wrong with the whole standard picture. Moreover, since the cosmological constant problem is a clash between two different branches of physics, gravitational physics and particle physics, we should not immediately assume that it is the particle physics side which needs addressing. Rather, the indications of particle physics (actually the better understood of the two) could well be correct, with its contribution to Λ actually being as big as it would appear to be, with the problem then having to lie on the gravitational side instead. Indeed, the very existence of the Friedmann evolution equation fine-tuning problem could itself be an indicator that it is in fact the gravitational side which is at fault. Thus, in the following we shall explicitly explore the implications for cosmology of Λ actually being a very big rather than a very small quantity, and, with attempts to quench Λ not having been successful thus far, we instead turn the issue around and explore below whether it is possible for cosmology to accommodate a large unquenched Λ instead. And, in particular, since the insertion of a large Λ into Eq. (1) does lead to such disastrous consequences (such as a $q(t_0)$ of order -10^{60} to -10^{120}), we shall thus consider the possibility that it is the standard Friedmann evolution equation which is in need of modification. And as we shall see, its replacement below by the cosmological evolution equation associated with the alternate conformal gravity theory will actually lead us to a cosmology (conformal cosmology) in which all of the above difficulties are naturally

and quite readily resolved.

To motivate the analysis of conformal cosmology which we will present below, we note that if we are indeed not going to quench Λ itself, then since it is actually $\Omega_\Lambda(t)$ which is its measurable consequence for cosmology (e.g. $q(t) = (n/2 - 1)\Omega_M(t) - \Omega_\Lambda(t)$), we must instead quench the amount of gravitation which a given cosmological source produces, i.e. we must instead quench G , so that instead of being controlled by the low energy Cavendish experiment G , cosmology needs to instead be controlled by an altogether smaller G_{eff} , with $\Omega_\Lambda(t)$ then being replaced by an effective $\bar{\Omega}_\Lambda(t) = 8\pi G_{eff}\Lambda/3cH^2(t)$, a quantity which would then be of order one today for an appropriately small enough G_{eff} . In such a case ordinary matter would then also have a quenched coupling to cosmic evolution with $\Omega_M(t)$ then being replaced by the effective $\bar{\Omega}_M(t) = 8\pi G_{eff}\rho_M(t)/3c^2H^2(t)$, a quantity which would (with the standard $\rho_M(t_0)$) now have to be of order 10^{-60} , with there then being no current era cosmic coincidence. Thus, since the $\Omega_\Lambda(t)/\Omega_M(t)$ ratio itself is actually independent of G (so that it can generically be written as T_V^4/T^4 where we set $\rho_M(t) = \sigma T^4$ for illustrative purposes), this ratio is also given as $\bar{\Omega}_\Lambda(t)/\bar{\Omega}_M(t)$, with the non-quenching of Λ then not leading us to a current era in which $c\Lambda$ is of order $\rho_M(t_0)$. Hence in the following we will seek to quench not the energy content of the universe but rather its effect on cosmic evolution. Moreover, continuing in this same vein, we additionally note that with the Friedmann equation initial condition fine-tuning problem itself deriving from the presence of an initial big bang singularity, this fine-tuning problem would also be avoided if there were to simply be no such initial singularity in the first place. Thus in the following we shall expressly look for theories in which the cosmological G_{eff} is not only altogether smaller than the standard G , but in which it is also even of the opposite (viz. gravitationally repulsive) sign. With such a small, negative G_{eff} both of the two aspects of the cosmological constant problem can then readily be resolved.

In order to see what would be needed of a theory which could potentially solve the cosmological constant problem this way, it turns out to be very instructive to first examine the problem within the context of the flat (negligible $\Omega_k(t)$) inflationary universe paradigm [6]. Thus solving Eq. (1) in the effectively flat Robertson-Walker phase which is to follow an inflationary de Sitter era, i.e. solving for $\rho_M(t) = B/R^3(t)$ (where $B > 0$), $k = 0$ and $\Lambda > 0$, yields $R(t) = (B/c\Lambda)^{1/3} \sinh^{2/3}(3D^{1/2}t/2)$ where $D = 8\pi G\Lambda/3c$, so that

$$\Omega_M(t) = \text{sech}^2(3D^{1/2}t/2), \quad \Omega_\Lambda(t) = D/H^2(t) = \tanh^2(3D^{1/2}t/2). \quad (4)$$

While an evolution such as this still possesses the above initial condition fine-tuning problem for a current era $\Omega_M(t_0) \simeq \Omega_\Lambda(t_0)$ (since the initial $\Omega_M(t=0) + \Omega_\Lambda(t=0)$ is still equal to one), nonetheless, as regards the magnitude of the actual contribution of the cosmological constant to cosmic evolution, we see that no matter how large or small Λ itself might be, the quantity $\Omega_\Lambda(t)$ itself always has to be less than or equal to one, being given by the nicely bounded $\tanh^2(3D^{1/2}t/2)$ form at all times. Thus given only a choice of sign for Λ , i.e. given only that the sign is in fact the one associated with standard model acceleration rather than deceleration, we see that the evolution associated with Eq. (4) then actually keeps the magnitude of the contribution of Λ to cosmic evolution under control in each and every epoch with $\Omega_\Lambda(t)$ always being smaller than one, and thus never being able to be of order 10^{60} even if Λ is in fact a particle physics sized scale. (Of course in such a case the ensuing Hubble parameter would have to readjust and be orders of magnitude different from

the currently measured one.) Cosmic acceleration thus appears to possess the germ of a possible solution to the cosmological constant problem within it in that it can (in principle at least) control $\Omega_\Lambda(t)$. Moreover, the functional dependence $\Omega_\Lambda(t) = \tanh^2(3D^{1/2}t/2)$ given above would still be obtained no matter what the actual numerical value of the parameter D , i.e. no matter what the value of G . Thus if we were to replace G by some smaller G_{eff} in Eq. (1), we would still obtain a bounded form for $\bar{\Omega}_\Lambda(t) = 8\pi G_{eff}\Lambda/3cH^2(t)$.

Additionally, we also note that if as well as change the magnitude of G we were also to even change its sign, so that G_{eff} would then be negative rather than positive, there would then (as already noted earlier) no longer be any initial singularity constraint. Thus solving the standard $k = 0$ theory with the wrong sign for G in Eq. (1), but with Λ now negative (so that $\Omega_\Lambda(t)$ is still positive), then yields $R(t) = (-B/c\Lambda)^{1/3} \cosh^{2/3}(3D^{1/2}t/2)$ where $D = 8\pi(-G)(-\Lambda)/3c$, so that

$$\Omega_M(t) = -\operatorname{cosech}^2(3D^{1/2}t/2), \quad \Omega_\Lambda(t) = \coth^2(3D^{1/2}t/2). \quad (5)$$

Now, instead of being equal to unity at $t = 0$, the initial $\Omega_M(t = 0)$ is instead infinite (since the now non-singular initial $\dot{R}(t = 0)$ vanishes rather diverges), and with the initial $\Omega_\Lambda(t = 0)$ being similarly infinite, initial conditions would no longer need to be fine-tuned in order to have the associated cosmology evolve into any particular current era value for $\Omega_M(t_0)$ or $\Omega_\Lambda(t_0)$. This change in sign for G thus releases us from the initial $\Omega_M(t = 0) + \Omega_\Lambda(t = 0) = 1$ constraint and enables us to resolve the standard model fine-tuning problem. In fact, that such a change in sign for G would immediately solve the cosmological flatness problem was already noted quite some time ago [7] within the context of a $\Lambda = 0$ conformal cosmology, where it was pointed out that once $\Omega_M(t)$ is negative (something which is the case in conformal gravity), early universe quantities that would have had to cancel with incredible precision in Eq. (1) no longer need do so. With a negative cosmological G_{eff} thus readily solving the cosmological initial condition problem, it is the purpose of the present paper to show how a small such negative G_{eff} will naturally (i.e. for a continuous, non fine-tuned range of parameters) emerge in the alternate conformal cosmology case, to thus not only permit conformal gravity to then readily control the contribution of Λ to cosmology, but to in fact do so in a way which will enable a perfectly conventional $\rho_M(t_0)$ to contribute to cosmology according to the non cosmic coincidence $\bar{\Omega}_M(t) = O(10^{-60})$ value, a value which, despite its initial appearance, will nonetheless not in fact turn out to be in conflict with observation. And since we have seen that there are some potentially useful generic aspects associated with the inflationary universe model, in order to be able to take advantage of them we turn now to a purely kinematic, model independent study of de Sitter geometry, one which will prove to be very instructive.

II. ESSENCE OF THE SOLUTION TO THE COSMOLOGICAL CONSTANT PROBLEM

Guided by the discussion given above, it is very instructive to analyze de Sitter geometry in a purely kinematic way which requires no commitment to any particular dynamical equation of motion [8], neither that of conformal gravity nor that of the standard model either for that matter. Specifically, suppose we know only that a given geometry is maximally 4-symmetric, i.e. that its Riemann tensor is given by

$$R^{\lambda\rho\sigma\nu} = \alpha(g^{\sigma\rho}g^{\lambda\nu} - g^{\nu\rho}g^{\lambda\sigma}). \quad (6)$$

For such a geometry contraction then yields the kinematic relation

$$R^{\mu\nu} - g^{\mu\nu}R^\sigma{}_\sigma/2 = 3\alpha g^{\mu\nu}, \quad (7)$$

a relation which reduces to

$$\dot{R}^2(t) + kc^2 = \alpha c^2 R^2(t) \quad (8)$$

when expressed in Robertson-Walker coordinates.² On defining $\Omega_\Lambda(t) = \alpha c^2 R^2(t)/\dot{R}^2(t)$ we obtain $-q(t) = \Omega_\Lambda(t) = 1 - \Omega_k(t)$, with $R(t)$, $q(t)$ and $\Omega_\Lambda(t)$ then being found [8] to be given by

$$\begin{aligned} R(t, \alpha < 0, k < 0) &= (k/\alpha)^{1/2} \sin((- \alpha)^{1/2} ct), \\ R(t, \alpha = 0, k < 0) &= (-k)^{1/2} ct, \\ R(t, \alpha > 0, k < 0) &= (-k/\alpha)^{1/2} \sinh(\alpha^{1/2} ct), \\ R(t, \alpha > 0, k = 0) &= R(t = 0) \exp(\alpha^{1/2} ct), \\ R(t, \alpha > 0, k > 0) &= (k/\alpha)^{1/2} \cosh(\alpha^{1/2} ct), \end{aligned} \quad (9)$$

and

$$\begin{aligned} \Omega_\Lambda(t, \alpha < 0, k < 0) &= -q(t, \alpha < 0, k < 0) = -\tan^2((- \alpha)^{1/2} ct), \\ \Omega_\Lambda(t, \alpha = 0, k < 0) &= -q(t, \alpha = 0, k < 0) = 0, \\ \Omega_\Lambda(t, \alpha > 0, k < 0) &= -q(t, \alpha > 0, k < 0) = \tanh^2(\alpha^{1/2} ct), \\ \Omega_\Lambda(t, \alpha > 0, k = 0) &= -q(t, \alpha > 0, k = 0) = 1, \\ \Omega_\Lambda(t, \alpha > 0, k > 0) &= -q(t, \alpha > 0, k > 0) = \coth^2(\alpha^{1/2} ct) \end{aligned} \quad (10)$$

in all of the various possible cases.³ As we thus see, when the parameter α is positive (this actually being the case in the conformal gravity theory we study below where $\alpha = 8\pi G_{eff}\Lambda/3c^3$ is greater than zero), each associated solution corresponds to an accelerating universe (only the $\alpha < 0$ anti de Sitter universe decelerates), and that in each such $\alpha > 0$ universe $\Omega_\Lambda(t, \alpha > 0)$ eventually reaches one no matter how big the parameter α might be, and independent in fact of whether or not G even appears in the cosmological evolution equations at all.⁴ Moreover, while the positive spatial 3-curvature $\Omega_\Lambda(t, \alpha > 0, k > 0)$ will

²While Eq. (8) is purely kinematic, in the following it will turn out to be crucial that the dynamical evolution equations explicitly reduce to it whenever the $\Omega_M(t)$ and $\bar{\Omega}_M(t)$ terms are negligible.

³The $\alpha > 0$ de Sitter geometry thus contains all three Robertson-Walker geometries and not just the $k = 0$ one, with it being the $k < 0$ one which will turn out to be the most consequential below.

⁴I.e. no matter how large α might be, the Hubble parameter always adjusts itself to be accordingly large so that $\Omega_\Lambda(t = \infty, \alpha > 0)$ is then equal to one - for instance in the familiar flat $k = 0$ case $\dot{R}(t)/R(t) = \alpha^{1/2}c$ at all times.

only come down to one at very late times, quite remarkably, the negative spatial 3-curvature $\Omega_\Lambda(t, \alpha > 0, k < 0)$ will be bounded between zero and one at all times, no matter how large α might be. Thus unlike the $\alpha < 0$ case where $\Omega_\Lambda(t)$ is unbounded, we see that when α is greater than zero (the accelerating situation), $\Omega_\Lambda(t)$ will either be bounded at all times or approach a bound at late times, and that additionally, even in the $\alpha = 0$ case, the 3-curvature dominated $\Omega_\Lambda(t, \alpha = 0, k < 0)$ will also be bounded as well. Thus we see first, that given enough time, any accelerating de Sitter cosmology (of any k) will, without any fine tuning at all, always eventually quench the contribution of a cosmological constant to cosmology no matter how large α might be, and even no matter what form the underlying gravitational theory might take. Second, in the particular $k < 0$ case which will prove to be the one relevant to conformal gravity itself, $\Omega_\Lambda(t, \alpha > 0, k < 0)$ will lie below one at all times, both early and late, and thus be bounded even at non-asymptotic times. And, third, since $\Omega_\Lambda(t, \alpha = 0, k < 0)$ is zero, all of the $k < 0, \alpha \geq 0$ cosmologies will be either curvature ($R(t) \sim t$) or cosmological constant ($R(t) \sim e^t$) dominated ones in which $0 \leq \Omega_\Lambda(t) \leq 1$ at all times, to thus not only nicely constrain the contribution of the vacuum energy density to cosmology, but to also put it precisely into the range required by the supernovae data. We thus distinguish between quenching Λ and quenching $\Omega_\Lambda(t)$, while noting that only the latter quenching is actually required by known cosmological observations. Thus, the parameter α need not itself be small, and in fact the larger it is, the faster $\Omega_\Lambda(t, \alpha > 0)$ will then approach its asymptotic bound, with the very same cosmic acceleration which has served to so exacerbate the cosmological constant problem thus potentially leading to its resolution. Having now described the essence of a possible solution to the cosmological constant problem, we turn now to a analysis of conformal gravity itself where this above bounding mechanism will be found to naturally appear (even as the very presence of ordinary $\rho_M(t) > 0$ matter obliges the geometry to be Robertson-Walker rather than de Sitter), with a small, negative G_{eff} being found to readily emerge and with the Cavendish experiment value for G no longer so sharply constraining cosmology.

III. SOLUTION TO THE COSMOLOGICAL CONSTANT PROBLEM

In attempting to depart from standard second order gravity (something we would appear to have to do if we are indeed going to replace G by an appropriate G_{eff}), even within the confines of covariant, pure metric based theories of gravity, we immediately realize that initially the choice is vast, since we can in principle consider covariant theories based on derivative functions of the metric of arbitrarily high order. However, within this infinite family of higher order derivative gravitational theories, one of them is immediately singled out, namely conformal gravity, a fully coordinate invariant gravitational theory which possesses an additional symmetry not enjoyed by the standard theory (viz. invariance under any and all local conformal stretchings $g_{\mu\nu}(x) \rightarrow \Omega^2(x)g_{\mu\nu}(x)$ of the geometry), a theory which consequently has as its uniquely allowed gravitational action the Weyl action

$$I_W = -\alpha_g \int d^4x (-g)^{1/2} C_{\lambda\mu\nu\kappa} C^{\lambda\mu\nu\kappa} \quad (11)$$

where $C^{\lambda\mu\nu\kappa}$ is the conformal Weyl tensor [9] and where α_g is a purely dimensionless gravitational coupling constant. Conformal gravity is thus a gravitational theory which possess

no fundamental scale (and thus no intrinsic G or fundamental Λ) at all, and is thus a theory which can immediately lead to a cosmology which is free of intrinsic scales at sufficiently high enough temperatures. As such, conformal gravity emerges as a potential gravitational analog of the Weinberg-Salam-Glashow electroweak theory, with it immediately being suggested [10,11] that in it Newton's constant G might be generated as a low energy effective parameter in much the same manner as Fermi's constant G_F is generated in the electroweak theory, with G as measured in a low energy Cavendish experiment then indeed nicely being decoupled from the hot early universe. However, it turns out that the low energy limit of conformal gravity need not emerge in precisely this fashion since [12] it is not in fact necessary to spontaneously break the conformal gravity action down to the Einstein-Hilbert action (i.e. down to the standard theory equations of motion). Rather [12], it is only necessary to obtain the solutions to those equations in the kinematic region (viz. solar system distance scales) where those standard solutions have been tested. Thus, as had been noted by Eddington [13] already in the very early days of relativity, the standard gravity vacuum Schwarzschild solution is just as equally a vacuum solution to higher derivative gravity theories also, since the vanishing of the Ricci tensor entails the vanishing of its derivatives as well. And indeed, with variation of the conformal gravity action leading to the equation of motion [14]

$$(-g)^{-1/2}\delta I_W/\delta g_{\mu\nu} = -2\alpha_g W^{\mu\nu} = -T^{\mu\nu}/2 \quad (12)$$

where $W^{\mu\nu}$ is given by

$$\begin{aligned} W^{\mu\nu} = & g^{\mu\nu}(R^\alpha{}_\alpha)^{;\beta}{}_{;\beta}/2 + R^{\mu\nu;\beta}{}_{;\beta} - R^{\mu\beta;\nu}{}_{;\beta} - R^{\nu\beta;\mu}{}_{;\beta} - 2R^{\mu\beta}R^\nu{}_\beta + g^{\mu\nu}R_{\alpha\beta}R^{\alpha\beta}/2 \\ & - 2g^{\mu\nu}(R^\alpha{}_\alpha)^{;\beta}{}_{;\beta}/3 + 2(R^\alpha{}_\alpha)^{;\mu;\nu}/3 + 2R^\alpha{}_\alpha R^{\mu\nu}/3 - g^{\mu\nu}(R^\alpha{}_\alpha)^2/6, \end{aligned} \quad (13)$$

and where $T^{\mu\nu}$ is the associated energy-momentum tensor, we confirm immediately that the Schwarzschild solution is indeed a vacuum solution to conformal gravity despite the total absence of the Einstein-Hilbert action in the purely gravitational piece I_W of the conformal action. Standard gravity is thus seen to be only sufficient to give the standard Schwarzschild metric phenomenology but not at all necessary, with it thus being possible to bypass the Einstein-Hilbert action altogether as far as low energy phenomena are concerned.⁵ As we shall show in detail below, it is precisely this aspect of the theory which will lead us to a demarcation between the high and low energy regions which is very different from that

⁵Since higher derivative theories have different continuations to larger distances [12], we see that the standard weak gravity Schwarzschild solar system distance scale wisdom is compatible with many differing extrapolations to larger distances, with standard gravity giving only one particular possible such extrapolation. And indeed, it has been argued [15,12] that this is actually the origin of the dark matter problem, with standard gravity simply giving an unsatisfactory extrapolation to galactic distance scales and beyond. Interestingly, the conformal gravity extrapolation [16] has been found to provide for a satisfactory explanation of galactic rotation curve systematics without the need to introduce any galactic dark matter at all.

present in the electroweak case.⁶

While conformal gravity itself is indeed an old idea, almost as old as General Relativity itself in fact, it is only recently that its potential role in cosmology and astrophysics appears to have been emphasized, with it having been found capable [7,8,12] [15–19] of addressing so many of the problems (such as the flatness, horizon, universe age, cosmic repulsion and dark matter problems) which currently afflict standard gravity. As a fundamental theory, conformal gravity has as a motivation the desire to give gravity a local invariance structure and a dimensionless coupling constant (and thus power counting renormalizability), to thereby make it analogous to the three other fundamental interactions. And indeed, as stressed in [17], the local conformal symmetry invoked to do this then not only excludes the existence of any fundamental mass scales such as a fundamental cosmological constant, even after mass scales are induced by spontaneous breakdown of the conformal symmetry, the (still) traceless energy-momentum tensor then constrains any induced (and necessarily negative) cosmological constant term to be of the same order of magnitude as all the other terms in $T^{\mu\nu}$, neither smaller nor larger. Thus, unlike standard gravity, precisely because of its additional symmetry, conformal gravity has a great deal of control over the cosmological constant (essentially, with all mass scales - of gravity and particle physics both - being jointly generated by spontaneous breakdown of the scale symmetry, conformal gravity knows exactly where the zero of energy is), and it is our purpose now to show that it is this very control which then provides for both a natural solution to the cosmological constant problem and for a complete accounting of the new high z data.

In conformal cosmology the conformal matter action takes the form [7]

$$I_M = -\hbar \int d^4x (-g)^{1/2} [S^\mu S_\mu / 2 - S^2 R^\mu{}_\mu / 12 + \lambda S^4 + i\bar{\psi}\gamma^\mu(x)(\partial_\mu + \Gamma_\mu(x))\psi - gS\bar{\psi}\psi] \quad (14)$$

for generic massless scalar and fermionic fields, a matter action which, because of the selfsame underlying conformal symmetry, contains up to only second order derivative functions of the matter fields. In this matter action we have introduced a dimensionful scalar field $S(x)$ which is to serve to spontaneously break the conformal symmetry,⁷ and for illustrative purposes we shall first consider just one such scalar field, giving the extension to more than one field below. And while for simplicity we use a fundamental scalar field here, we anticipate that terms such as the λS^4 term will actually be generated via an effective Ginzburg-Landau theory in which $S(x)$ is to serve as a phase transition condensate order parameter, and in which the λS^4 term is to represent the necessarily negative vacuum energy density $V_{min}^{GL} = -g(T_V^2 - T^2)^2$ associated with the typical effective Ginzburg-Landau potential

⁶We would anyway expect there to have to be some difference between the ways the spontaneous breakdown mechanisms work in the two cases, since low energy gravity is not short range.

⁷The underlying conformal symmetry forces the sign of the required $-S^2 R^\mu{}_\mu / 12$ term in Eq. (14) to uniquely be negative, and as such it would at first sight appear that after spontaneous breakdown this term would then generate an effective low energy gravity theory which would be repulsive. However, as we will show below, it will turn out that this term will in fact only be operative cosmologically where such gravitational repulsion now appears welcome.

$V_{GL}(S, T) = gS^4 - 2g(T_V^2 - T^2)S^2$ at its $T < T_V$ ordered phase minimum. In such a case the effective $\Lambda \simeq V_{min}^{GL}$ would be negative, and would necessarily have to in fact be so in conformal gravity, since in a theory with an underlying conformal symmetry, the (dimensionful) vacuum energy density has to be zero identically in the $S(x) = 0$ scale invariant high temperature regime above all scale generating phase transitions where the presence of an exact, unbroken conformal symmetry ensures the absence of any fundamental scale. Thus unlike the situation in the standard theory, in the conformal theory the sign of Λ is actually known a priori, in fact explicitly being opposite to that assumed in the standard model. However, despite being negative (something we simulate below by taking the parameter λ in Eq. (14) to be negative), as we shall see, it will precisely be this particular sign which will actually lead to cosmic acceleration in the conformal theory.

For the above matter action the matter field equations of motion take the form

$$\begin{aligned} i\gamma^\mu(x)[\partial_\mu + \Gamma_\mu(x)]\psi - gS\psi &= 0, \\ S^\mu_{;\mu} + SR^\mu_\mu/6 - 4\lambda S^3 + g\bar{\psi}\psi &= 0, \end{aligned} \quad (15)$$

with the matter energy-momentum tensor being given by

$$\begin{aligned} T^{\mu\nu} = \hbar\{i\bar{\psi}\gamma^\mu(x)[\partial^\nu + \Gamma^\nu(x)]\psi + 2S^\mu S^\nu/3 - g^{\mu\nu}S^\alpha S_\alpha/6 - SS^{\mu;\nu}/3 \\ + g^{\mu\nu}SS^\alpha_{;\alpha}/3 - S^2(R^{\mu\nu} - g^{\mu\nu}R^\alpha_\alpha/2)/6 - g^{\mu\nu}\lambda S^4\}. \end{aligned} \quad (16)$$

Thus in the case where the scalar field acquires a non-zero vacuum expectation value (an expectation value which can always be rotated into a spacetime constant S_0 by an appropriate local conformal transformation), the entire energy-momentum tensor of the theory is found (for a perfect matter fluid $T_{kin}^{\mu\nu}$ of the fermions) to take the form

$$T^{\mu\nu} = T_{kin}^{\mu\nu} - \hbar S_0^2(R^{\mu\nu} - g^{\mu\nu}R^\alpha_\alpha/2)/6 - g^{\mu\nu}\hbar\lambda S_0^4; \quad (17)$$

with the complete solution to the scalar, fermionic and gravitational field equations of motion in a background Robertson-Walker geometry (viz. a geometry in which the Weyl tensor and $W^{\mu\nu}$ both vanish) then reducing [8] to just one relevant equation, namely

$$T^{\mu\nu} = 0, \quad (18)$$

a remarkably simple condition which immediately fixes the zero of energy, so that, and unlike the situation in the standard theory, the conformal theory does indeed know exactly where the zero of energy is. Moreover, not only does this condition fix the zero of energy, it even serves to fix the spatial curvature of the universe as well, with Eq. (18) itself reducing to the even simpler $T_{kin}^{\mu\nu} = 0$ at the earliest temperatures above all phase transitions, a condition which is then only satisfiable non-trivially [19] when $k < 0$, with the positive energy of ordinary matter being explicitly canceled by the negative gravitational energy associated with negative spatial curvature. Thus in the conformal theory the global topology of the universe is fixed once and for all before any cosmological phase transition ever occurs.

With the imposition of Eq. (18) enabling us to rewrite Eq. (17) in the form

$$\hbar S_0^2(R^{\mu\nu} - g^{\mu\nu}R^\alpha_\alpha/2)/6 = T_{kin}^{\mu\nu} - g^{\mu\nu}\hbar\lambda S_0^4, \quad (19)$$

we thus see that the evolution equation of conformal cosmology looks identical to that of standard gravity save only that the quantity $-\hbar S_0^2/12$ has replaced the familiar $c^3/16\pi G$. With this homogeneous and isotropic, global scalar field S_0 filling all space and acting cosmologically, we see that this change in sign compared with standard gravity leads to a cosmology in which gravity is globally repulsive rather than attractive. Because of this change in sign, conformal cosmology thus has no initial singularity (i.e. it expands from a finite minimum radius), and is thus precisely released from the standard big bang model fine-tuning constraints described earlier. Similarly, because of this change in sign the contribution of $\rho_M(t)$ to the expansion of the universe is now effectively repulsive, to (heuristically) mesh with the phenomenological high z data fits in which $\Omega_M(t)$ was allowed to go negative. Apart from a change in sign, we see that through S_0 there is also a change in the strength of gravity compared to the standard theory. It is this feature which will prove central to the solution to the cosmological constant problem which we present below.

Despite the fact that conformal gravity has now been found to be globally repulsive, nonetheless, it is important to note that in the conformal theory local solar system gravity can still be attractive; with it having been specifically found [12] that for a static, spherically symmetric source such as a star, the conformal gravity field equation of Eq. (12) reduces to a fourth order (i.e. not a second order) Poisson equation $\nabla^4 g_{00} = 3(T_0^0 - T_r^r)/4\alpha_g g_{00} \equiv -f(r)$, with solution $-g_{00}(r) = 1 - 2\beta^*/r + \gamma^*r$ where $\beta^* = \int dr f(r)r^4/12$ and $\gamma^* = -\int dr f(r)r^2/2$. With the coupling constant α_g in the Weyl action I_W simply making no contribution in highly symmetric cosmologically relevant geometries where $C^{\lambda\mu\nu\kappa}$ and $W^{\mu\nu}$ vanish, and with the sign of β^* being directly given by the sign of this thus cosmologically irrelevant α_g , we see that locally attractive and globally repulsive gravity are now decoupled and thus able to coexist. Local gravity is thus fixed by local sources alone, sources which are only gravitational inhomogeneities in the otherwise homogeneous global cosmological background, i.e. sources which are characterized by small, local variations in the background scalar field $S(x)$, variations which themselves are completely decoupled from the homogeneous, constant, cosmological background field S_0 itself. It is thus the distinction between homogeneity and inhomogeneity which provides the demarcation between local and global gravity, to thus now enable us to consider repulsive cosmologies which are not incompatible with the attractive gravity observed on solar system distance scales.

Given the equation of motion $T^{\mu\nu} = 0$, the conformal cosmology evolution equations are then found to take the form (on setting $\Lambda = \hbar\lambda S_0^4$)

$$\dot{R}^2(t) + kc^2 = -3c^3\dot{R}^2(t)(\Omega_M(t) + \Omega_\Lambda(t))/4\pi\hbar S_0^2 G \equiv \dot{R}^2(t)(\bar{\Omega}_M(t) + \bar{\Omega}_\Lambda(t)) \quad (20)$$

$$\bar{\Omega}_M(t) + \bar{\Omega}_\Lambda(t) + \Omega_k(t) = 1, \quad q(t) = (n/2 - 1)\bar{\Omega}_M(t) - \bar{\Omega}_\Lambda(t) \quad (21)$$

where Eq. (20) serves to define $\bar{\Omega}_M(t)$ and $\bar{\Omega}_\Lambda(t)$. As we see, precisely because the underlying conformal invariance has forced the conformal $T^{\mu\nu}$ to be of the standard second order form, Eq. (20) is found to be remarkably similar in form to Eq. (1), with conformal cosmology thus only containing familiar ingredients. As an alternate cosmology then, conformal gravity thus gets about as close to standard gravity as it is possible for an alternative to get while nonetheless still being different. In fact the two sets of evolution equations are actually completely equivalent to each other save only in the replacement of the standard attractive G by a repulsive effective cosmological G_{eff} given by $-3c^3/4\pi\hbar S_0^2$, a replacement

which precisely gives the conformal G_{eff} the very structure desired above of an effective cosmological G_{eff} which is to indeed solve the cosmological constant problem. With such a G_{eff} we also see that the larger S_0 the smaller the resulting contribution of Λ to cosmic evolution, with conformal gravity thus being able to accommodate a far larger Λ than the standard theory.

In order to see whether conformal gravity can actually accommodate the new supernovae data with such a G_{eff} , it is necessary to analyze the solutions to Eq. (20). Such solutions are readily obtained [8], and can be classified according to the signs of λ and k (and even though we have indicated above that λ and k are to both be negative in the conformal case, nonetheless for completeness we explore all possible cases here). In the simpler to treat high temperature era where $\rho_M(t) = A/R^4 = \sigma T^4$ the complete family of solutions is given as

$$\begin{aligned} R^2(t, \alpha < 0, k < 0) &= k(1 - \beta)/2\alpha + k\beta \sin^2((- \alpha)^{1/2}ct)/\alpha, \\ R^2(t, \alpha = 0, k < 0) &= -2A/k\hbar c S_0^2 - kc^2t^2, \\ R^2(t, \alpha > 0, k < 0) &= -k(\beta - 1)/2\alpha - k\beta \sinh^2(\alpha^{1/2}ct)/\alpha, \\ R^2(t, \alpha > 0, k = 0) &= (-A/\hbar\lambda c S_0^4)^{1/2} \cosh(2\alpha^{1/2}ct), \\ R^2(t, \alpha > 0, k > 0) &= k(1 + \beta)/2\alpha + k\beta \sinh^2(\alpha^{1/2}ct)/\alpha, \end{aligned} \quad (22)$$

where we have introduced the parameters $\alpha = -2\lambda S_0^2$ and $\beta = (1 - 16A\lambda/k^2\hbar c)^{1/2}$. Similarly the associated deceleration parameters take the form

$$\begin{aligned} q(t, \alpha < 0, k < 0) &= \tan^2((- \alpha)^{1/2}ct) - 2(1 - \beta)\cos(2(- \alpha)^{1/2}ct)/\beta \sin^2(2(- \alpha)^{1/2}ct), \\ q(t, \alpha = 0, k < 0) &= -2A/k^2\hbar c^3 S_0^2 t^2, \\ q(t, \alpha > 0, k < 0) &= -\tanh^2(\alpha^{1/2}ct) + 2(1 - \beta)\cosh(2\alpha^{1/2}ct)/\beta \sinh^2(2\alpha^{1/2}ct), \\ q(t, \alpha > 0, k = 0) &= -1 - 2/\sinh^2(2\alpha^{1/2}ct), \\ q(t, \alpha > 0, k > 0) &= -\coth^2(\alpha^{1/2}ct) - 2(1 - \beta)\cosh(2\alpha^{1/2}ct)/\beta \sinh^2(2\alpha^{1/2}ct). \end{aligned} \quad (23)$$

Now while Eq. (22) yields a variety of temporal behaviors for $R(t)$, it is of great interest to note that every single one of them begins with $\dot{R}(t=0)$ being zero rather than infinite (with $\bar{\Omega}_M(t=0)$ and $\bar{\Omega}_\Lambda(t=0)$ then both being infinite regardless of what their current era values might be), and that each one of the solutions in which λ is negative (viz. $\alpha > 0$) is associated with a universe which permanently expands (only the $\lambda > 0$ solution can recollapse, with conformal cosmology thus correlating the long time behavior of $R(t)$ with the sign of λ rather than with the sign of k). We thus need to determine the degree to which the permanently expanding universes have by now already become permanently accelerating.

To this end we note first from Eq. (23) that with β being greater than one when λ is negative, both the $\alpha > 0, k < 0$ and the $\alpha > 0, k = 0$ cosmologies are in fact permanently accelerating ones no matter what the values of their parameters. To explore the degree to which they have by now already become asymptotic, as well as to determine the acceleration properties of the $\alpha > 0, k > 0$ cosmology, we note that since each of the solutions given in Eq. (22) has a non-zero minimum radius, each associated $\alpha > 0$ cosmology has some very large but finite maximum temperature T_{max} given by

$$\begin{aligned} T_{max}^2(\alpha > 0, k < 0)/T^2(t, \alpha > 0, k < 0) &= 1 + 2\beta \sinh^2(\alpha^{1/2}ct)/(\beta - 1), \\ T_{max}^2(\alpha > 0, k = 0)/T^2(t, \alpha > 0, k = 0) &= \cosh(2\alpha^{1/2}ct), \\ T_{max}^2(\alpha > 0, k > 0)/T^2(t, \alpha > 0, k > 0) &= 1 + 2\beta \sinh^2(\alpha^{1/2}ct)/(\beta + 1), \end{aligned} \quad (24)$$

with all the permanently expanding ones thus necessarily being way below their maximum temperatures once given enough time. To obtain further insight into these solutions it is convenient to introduce an effective temperature according to $-c\hbar\lambda S_0^4 = \sigma T_V^4$. In terms of this T_V we then find that in all the $\lambda < 0$ cosmologies the energy density terms take the form

$$\begin{aligned}\bar{\Omega}_\Lambda(t) &= (1 - T^2/T_{max}^2)^{-1}(1 + T^2T_{max}^2/T_V^4)^{-1}, \\ \bar{\Omega}_M(t) &= -(T^4/T_V^4)\bar{\Omega}_\Lambda(t),\end{aligned}\tag{25}$$

where $(\beta - 1)/(\beta + 1) = T_V^4/T_{max}^4$ for the $k < 0$ case, and where $(\beta - 1)/(\beta + 1) = T_{max}^4/T_V^4$ for the $k > 0$ case. With β being greater than one, we find that for the $k > 0$ case T_V is greater than T_{max} , for $k = 0$ T_V is equal to T_{max} , and for $k < 0$ T_V is less than T_{max} , with the energy in curvature (viz. the energy in the gravitational field itself) thus making a direct contribution to the maximum temperature of the universe. Hence, simply because the temperature T_{max} is overwhelmingly larger than the current temperature $T(t_0)$ (i.e. simply because the universe has been expanding and cooling for such a long time now), we see that, without any fine tuning at all, in both the $k > 0$ and $k = 0$ cases (i.e. cases where $T_V \geq T_{max} \gg T(t_0)$), the quantity $\bar{\Omega}_\Lambda(t_0)$ is already at its asymptotic limit of one today, that $\bar{\Omega}_M(t_0)$ is completely suppressed, and that the deceleration parameter is given by $q(t_0) = -1$.

For the $\alpha > 0$, $k < 0$ case (the only $\alpha > 0$ case where T_V is less than T_{max} , with a large T_V thus automatically entailing an even larger T_{max}) a very different outcome is possible however. Specifically, since in this case the quantity $(1 + T^2T_{max}^2/T_V^4)^{-1}$ is always bounded between zero and one no matter what the relative magnitudes of T_V , T_{max} and $T(t)$, we see that as long as $T_{max} \gg T(t_0)$ (which would even have to be the case if $T_V \gg T(t_0)$), rather than having had to have already reached its asymptotic limit of one by now, the quantity $\bar{\Omega}_\Lambda(t_0)$ is instead only required to be bounded by it. And with it thus expressly being bounded from above, in the $k < 0$ case the current era value of $\bar{\Omega}_\Lambda(t_0)$ thus has to lie somewhere between zero and one today no matter how big or small T_V might be (and despite the fact that $\bar{\Omega}_\Lambda(t_0)$ is even infinite in the early universe); with the simple additional requirement that T_V also be very much greater than $T(t_0)$ (viz. large Λ) then entailing that $\bar{\Omega}_M(t_0)$ will yet again be completely suppressed in the current era. Moreover, $\bar{\Omega}_\Lambda(t_0)$ will take a typical value of one half should the value of the quantity $T^2(t_0)T_{max}^2/T_V^4$ currently be close to one. Values of $\bar{\Omega}_\Lambda(t_0)$ not merely less than one but even appreciably so are thus readily achievable in the $k < 0$ case for a continuous range of temperature parameters which obey $T_{max} \gg T_V \gg T(t_0)$ without the need for any fine-tuning at all. And with the current era $\bar{\Omega}_M(t_0)$ being completely suppressed and with the early universe $\bar{\Omega}_M(t = 0)$ being infinite, there is thus no cosmic coincidence fine-tuning problem either, with conformal cosmology being completely free of fine-tuning problems.

Noting from Eq. (24) that the temporal evolution of the $\alpha > 0$, $k < 0$ case is given by

$$\tanh^2(\alpha^{1/2}ct) = (1 - T^2/T_{max}^2)/(1 + T^2T_{max}^2/T_V^4),\tag{26}$$

we see that simply because of the fact that $T_{max} \gg T(t_0)$, the current era value of $\bar{\Omega}_\Lambda(t_0)$ is then given by the nicely bounded $\tanh^2(\alpha^{1/2}ct_0)$ form, i.e. given precisely by the form found in the model independent analysis of de Sitter space that was presented above. Additionally, in this case $\Omega_k(t_0)$ is then given by $\text{sech}^2(\alpha^{1/2}ct_0)$, with negative spatial curvature

then explicitly contributing to current era cosmology (and even doing so repulsively since $q(t_0) = (n/2 - 1)(1 + kc^2/\dot{R}^2(t_0)) - n\bar{\Omega}_\Lambda(t_0)/2$), with the current era universe not needing to yet be vacuum (viz. cosmological constant) dominated. Robertson-Walker conformal cosmology thus goes through three epochs, matter domination (where $q(t) \sim -\infty$), curvature domination ($R(t) \sim t$, $q(t) \sim 0$) and then finally vacuum domination ($R(t) \sim e^t$, $q(t) \sim -1$), with matter domination of the expansion rate being restricted to the very early universe (thus incidentally making it irrelevant for later epochs whether we take $\rho_M(t) = A/R^4$ or $\rho_M(t) = B/R^3$ in them), with the current era universe lying somewhere in or between the curvature and vacuum dominated phases. The universe can thus currently be in a mild linearly expanding phase, with it not needing to have to already be in a far more rapidly growing exponential one. Additionally, we also note that in the $\alpha > 0$, $k < 0$ case the Hubble parameter is given by

$$H(t) = \alpha^{1/2}c(1 - T^2(t)/T_{max}^2)/\tanh(\alpha^{1/2}ct), \quad (27)$$

with its current value thus obeying $-q(t_0)H^2(t_0) = \alpha c^2$ and with the current age of the universe being given by $t_0 H(t_0) = \text{arctanh}[(-q(t_0))^{1/2}]/(-q(t_0))^{1/2}$.⁸ Thus no matter what the explicit value of α the current era $q(t_0)$ always has to lie between zero and minus one with the Hubble parameter adjusting itself so as to be given by $H^2(t_0) = -\alpha c^2/q(t_0)$, and with the $-q(t)H^2(t)$ product being independent of time at all times $T(t) \ll T_{max}$. The quantity $\bar{\Omega}_\Lambda(t)H^2(t)$ thus asymptotes to αc^2 , a behavior quite reminiscent of that found in the inflationary universe which we exhibited earlier as Eq. (4). Thus when k is negative (as noted earlier this is actually the theoretically preferred choice in the conformal theory), we find that values of $\bar{\Omega}_\Lambda(t_0)$ less than one are then naturally achievable in the conformal theory, and with $\bar{\Omega}_\Lambda(t_0)$ not being able to be larger than one (no matter what the value of k in fact) once given only that the parameter α is in fact positive and that the universe is as old as it is, we see that a conformal cosmology universe solves the cosmological constant problem simply by living for a very long time.

Thus we see that in all three of the $\alpha > 0$ cases the simple requirement that $T_{max} \gg T(t_0)$, $T_V \gg T(t_0)$ ensures that $\bar{\Omega}_M(t_0)$ is completely negligible at current temperatures (it can thus only be relevant in the early universe), with the current era Eq. (20) then reducing to

$$\dot{R}^2(t) + kc^2 = \dot{R}^2(t)\bar{\Omega}_\Lambda(t) \quad (28)$$

to thus not only yield as a current era conformal cosmology what in the standard theory could only possibly occur as a very late one, but to also yield one which enjoys all the nice purely kinematic properties of a de Sitter geometry which we identified above. Since studies of galaxy counts indicate that the purely visible matter contribution to $\Omega_M(t_0)$ is of order one (actually of order 10^{-2} or so in theories in which dark matter is not considered), it follows from Eq. (20) that current era suppression of $\bar{\Omega}_M(t_0)$ will in fact be achieved if the conformal cosmology scale parameter S_0 is altogether larger than the inverse Planck

⁸The age t_0 is thus necessarily greater than $1/H(t_0)$ ($t_0 = 1/H(t_0)$ when $\alpha = 0$), with it taking the typical value $t_0 = 1.25/H(t_0)$ when $q(t_0) = -1/2$. Thus as already noted in [18,8] conformal cosmologies have no universe age problem.

length L_{PL}^{-1} , a condition which is naturally imposable (i.e. for a continuous, non fine-tuned, range of parameters of the theory) and which is precisely compatible with a large rather than a small S_0 . Comparison with Eq. (1) shows that a current era $\lambda < 0$ conformal cosmology looks exactly like a low mass standard model cosmology, except that instead of $\Omega_M(t_0)$ being negligibly small (something difficult to understand in the standard theory) it is $\bar{\Omega}_M(t_0) = -3\Omega_M(t_0)/4\pi S_0^2 L_{PL}^2$ which is negligibly small instead ($\Omega_M(t_0)$ itself need not actually be negligible in conformal gravity - rather, it is only the contribution of $\rho_M(t)$ to the evolution of the current universe which needs be small). Hence, we see that the very essence of our work is that the same mechanism which causes $\bar{\Omega}_\Lambda(t_0)$ to be of order one today, viz. a large rather than a small $\Lambda = \hbar\lambda S_0^4$, serves at the same time, and without any fine tuning, to cause $\bar{\Omega}_M(t_0)$ to decouple from current era cosmology, and to thus not have to take a value anywhere near close in magnitude to that taken by $\bar{\Omega}_\Lambda(t_0)$. Thus to conclude we see that when λ is negative, that fact alone is sufficient to automatically drive us into the narrow (supernovae data compatible) $\bar{\Omega}_M(t_0) = 0$, $0 \leq \bar{\Omega}_\Lambda(t_0) \leq 1$ window,⁹ with the current era $\bar{\Omega}_\Lambda(t_0)$ being able to be less than (and even much less than) one in the negative spatial curvature case.

While we have thus shown that in conformal gravity it is indeed possible to completely control the contribution of the cosmological constant to cosmic evolution without the need for any fine-tuning, for practical applications of the theory it would be helpful if we could constrain the allowed values of the parameter α which controls the actual bounded $\tanh(\alpha^{1/2}ct_0)$ value which $\bar{\Omega}_\Lambda(t_0)$ takes. Additionally, we also need to extend our analysis to allow for the presence of more than one scalar field, and it is thus of interest to note that it is just such an extension which will actually enable us to provide the desired α constraint. Specifically, since conformal invariance is to be completely unbroken in the very earliest universe with the vacuum energy density $\Lambda = V_{min}^{GL}$ being zero at temperatures above the (highest) critical temperature T_V (a temperature which we recall is less than T_{max} in the $k < 0$ case), and since the $R^2(t, \alpha = 0, k < 0) = -2A/k\hbar c S_0^2 - kc^2 t^2$ scale factor has a non-zero minimum value even in the absence of any α (with Eq. (20) actually only admitting of negative k solutions when Λ is zero), we thus see that in the conformal theory there would be a maximum (negative curvature supported) temperature $T_{max}^2 \sim -k\hbar c S_0^2/2A$ even in the absence of spontaneous breakdown of the conformal symmetry, with the values of T_{max} and T_V thus being completely decoupled.¹⁰ In order to actually have a non-zero S_0 above all critical temperatures, i.e. to actually have a non-zero S_0 even without any spontaneous breakdown, we must thus introduce a fundamental, non-dynamical, conformally coupled "urfeld" scalar field $S(x)$ which fills all space. For such a field, the solution to its

⁹Given the similarity of Eqs. (1) and (20), phenomenological high z data fits are thus fits to both standard and conformal gravity, with conformal gravity nicely locking into an $\bar{\Omega}_M(t_0) = 0$ window in which a perfectly normal $\rho_M(t)$ makes a completely negligible contribution to cosmic expansion.

¹⁰In passing we note that in a $\Lambda = 0$ conformal cosmology $q(t)$ is found [18] to be given by $q(t) = [1 - T_{max}^2/T^2(t)]^{-1}$, with its current era value thus being negligibly small. Thus, as had actually been noted well in advance of the recent high z supernovae data, $\Lambda = 0$ conformal cosmology already possesses a repulsion not present in a $\Lambda = 0$ standard model (where $q(k = 0, t_0) = 1/2$).

field equation $S^\mu{}_\mu + SR^\mu{}_\mu/6 = 0$ in a maximally 3 symmetric background geometry would have to depend only on time, with a purely time dependent conformal transformation then bringing it to a constant value S_0 , with a resetting of the time bringing the simultaneously conformally transformed geometry back to the standard Robertson-Walker form, and with $R^2(t, \alpha = 0, k < 0)$ then being given by Eq. (22) just as required. In addition to this urfeld we introduce at a much lower temperature a second scalar field, one associated with a typical particle physics symmetry breaking vacuum expectation value S'_0 where S'_0 is thus altogether smaller than S_0 . While both of these fields will conformally couple to the Ricci scalar in Eq. (14), it will be the much larger early universe urfeld S_0 which will dominate G_{eff} (so that our requirement that S_0 be altogether larger than L_{PL}^{-1} is thus readily satisfiable even while S'_0 itself is typically much smaller), while it will be the particle physics one which will generate $-c\Lambda = -c\hbar\lambda'S_0'^4 = \sigma T_V^4$. In the presence of both of these fields we can continue to use the previous formalism provided we identify the effective λ parameter in Eqs. (14) and (17) according to $\lambda = \lambda'S_0'^4/S_0^4 \ll \lambda'$. With such a choice T_{max}/T_V will typically be as large as S_0/S'_0 with the parameter $\alpha c^2 = -2\lambda S_0^2 = -2\lambda'S_0'^4/S_0^2$ then being much smaller than $-2\lambda'S_0'^2$. Thus with q_0 being given by $-\tanh^2(\alpha^{1/2}ct_0) = -(1 + T^2(t_0)T_{max}^2/T_V^4)^{-1}$, the quantity $\alpha^{1/2}ct_0$ can thus readily be small enough to prevent q_0 from already being at its asymptotic value of minus one, with a really big T_{max} bringing $q(t_0)$ very close to its curvature dominated value of zero.

Now while we have already indicated that $k < 0$ spatial curvature is theoretically preferred in conformal gravity, we also note that there is even observational support for this value [16], support obtained from an at first highly unlikely source, namely galactic rotation curve data. Recalling that in conformal gravity the metric outside of a static spherically symmetric source such as a star is given by $-g_{00}(r) = 1 - 2\beta^*/r + \gamma^*r$, we see that in the conformal theory the departure from Newton is found to be given by a potential that actually grows (linearly) with distance. Hence, unlike the situation in standard gravity, in conformal gravity it is not possible to ever neglect the matter exterior to any region of interest, with the rest of the universe (viz. the Hubble flow) then also contributing to galactic motions (i.e. a test particle in a galaxy not only samples the local galactic gravitational field, it also samples that of the global Hubble flow as well). And indeed, it was found [16] that the effect on galaxies of the global Hubble flow was to generate an additional linear potential with a universal coefficient given by $\gamma_0/2 = (-k)^{1/2}$, i.e. one which is generated explicitly by the negative scalar curvature of the universe¹¹ (heuristically, the repulsion associated with negative scalar curvature pushes galactic matter deeper into any given galaxy, an effect which an observer inside that galaxy interprets as attraction), with conformal gravity then being found able to give an acceptable accounting of galactic rotation curve systematics (in the data fitting γ_0 is numerically found to be given by $3.06 \times 10^{-30} \text{ cm}^{-1}$, i.e. to be explicitly given by a cosmologically significant length scale) without recourse to dark matter at

¹¹Essentially, under the general coordinate transformation $r = \rho/(1 - \gamma_0\rho/4)^2$, $t = \int d\tau/R(\tau)$, a static, Schwarzschild coordinate observer in the rest frame of a given galaxy recognizes the (conformally transformed) comoving Robertson-Walker metric $ds^2 = \Omega(\tau, \rho)[c^2d\tau^2 - R^2(\tau)(d\rho^2 + \rho^2d\Omega)/(1 - \rho^2\gamma_0^2/16)^2]$ (where $\Omega(\tau, \rho) = (1 + \rho\gamma_0/4)^2/R^2(\tau)(1 - \rho\gamma_0/4)^2$) as being conformally equivalent to the metric $ds^2 = (1 + \gamma_0r)c^2dt^2 - dr^2/(1 + \gamma_0r) - r^2d\Omega$.

all. We thus identify an explicit imprint of cosmology on galactic rotation curves, recognize that it is its neglect which may have led to the need for dark matter, and for our purposes here confirm that k is indeed negative.¹² Conformal cosmology thus leads us directly to $\bar{\Omega}_M(t_0) = 0$, $\bar{\Omega}_\Lambda(t_0) = \tanh^2(\alpha^{1/2}ct_0)$, and would thus appear to lead us naturally right into the region favored by the new high z data.

As regards the role played by negative Λ in conformal cosmology, we recall that the effect of elementary particle physics phase transitions is to lead to a vacuum energy density V_{min}^{GL} which is typically expected to be negative rather than positive since each one of the many particle physics phase transitions acts to lower the vacuum energy density some more as the universe cools. Once given such a negative Λ its effect on cosmic evolution then depends on the sign of the effective G . Thus, for repulsive conformal cosmology, negative Λ translates into positive $\bar{\Omega}_\Lambda(t)$ and thus to cosmic acceleration, whereas for the attractive standard cosmology it translates into negative $\Omega_\Lambda(t)$, to then not lead to any cosmic acceleration at all. Thus added to the challenges faced by the standard theory is the need to explain not only why Λ should be small, but also to explain why it should also not in fact be negative, with the very fact of cosmic acceleration providing some support for the central theme of our work, namely that cosmologically, the effective G is in fact negative.

As regards such an effective negative cosmological G , we note that are essentially two primary arguments which have in the past supported the contrary, G positive, cosmological position, namely the current value of $\Omega_M(t_0)$ and big bang nucleosynthesis. However, of these two, the $\Omega_M(t_0)$ argument now has to be discounted. Specifically, with earlier (i.e. pre high z) data having led to a current value of $\Omega_M(t_0) = 8\pi G\rho_M(t_0)/3c^2H^2(t_0)$ which was tantalizingly close to one (provided one included dark matter that is), it strongly suggested that cosmology was indeed normalized to the gravitational constant G , with cosmological theory otherwise having to explain this closeness as an accident. And, indeed, the great appeal of inflation was that it provided a rationale for having $\Omega_M(t_0)$ be close to one today by having $\Omega_M(t)$ be identically equal to one in each and every epoch. However, with the new high z data, we now know that $\Omega_M(t_0)$ is unambiguously less than one, and more, that it will get ever smaller as the universe continues to accelerate. Thus, for observers sufficiently far enough into the future $\Omega_M(t)$ will be nowhere near one, with its current closeness to one being only an artifact of the particular epoch in which current observers happen to be making observations (and of course without dark matter, by itself known explicitly detected luminous matter only yields for $\Omega_M(t_0)$ a value which is actually a few orders of magnitude or so below one today).

With regard to nucleosynthesis, we note that, in principle, it only requires that the universe had once been hot enough to have been able to trigger nuclear reactions, with it not being at all necessary that even earlier there had been an altogether hotter ($G > 0$ induced) big bang phase. And indeed, it has been found [21–23] that since the universe has been expanding and cooling for such a very long time now, conformal cosmology is also

¹²Given the presence of the imprint of such a cosmological scale on galaxies, it thus becomes necessary (see also [20,19] for related discussion) for dark matter models to equally produce such a scale, something which may not be all that easy in standard flat $k = 0$ models where no curvature scale is available.

capable of having once been hot enough to have undergone nucleosynthesis (even without a big bang); with the latest calculations [23] yielding the requisite amount of helium as well as the metallicity which is explicitly seen in population II stars,¹³ with the inability [21–23] of conformal cosmology to yield a sufficient amount of deuterium being its only outstanding nucleosynthesis problem. Now, as regards the production of deuterium, we note that while it is generally thought difficult to produce post-primordially, this is not quite the case, as it is actually fairly easy to both produce and then retain deuterium by spallation or fragmentation of light nuclei [24–26], particularly if the spallation is pre rather than post galactic.¹⁴ In fact the problem is then not one of an underproduction of deuterium, but rather of an overproduction of the other light elements. However, as noted by Epstein [26], if the spallation is to also take place in the early universe with its onset occurring after the nucleosynthesis itself, then (i) in such a situation only hydrogen and helium interactions would be of any significance, with only $Z \leq 3$ nuclei then being producible, and (ii) that in such a case the high energies involved would serve to favor deuterium production over the lithium production which is favored at ordinary energies. In addition to this, the authors of [23], on having found the helium abundance in their nucleosynthesis calculations to be a rather sensitive function of the baryon to entropy ratio, have suggested that lithium production could also be suppressed if the spallation were to take place inhomogeneously with helium deficient clouds then spallating with helium rich ones. In such a case, deuterium would then be produced not during nucleosynthesis itself but some time afterwards just as inhomogeneities first begin to form in the universe.¹⁵ Since a theory for the growth of inhomogeneities in conformal cosmology has not yet been developed, unfortunately, it is not yet possible to currently provide a detailed analysis of this issue or assess its implications for conformal gravity, though the issue is of course of paramount importance for the conformal theory.

While nucleosynthesis continues to be the primary achievement of standard big bang cosmology, nonetheless, quite recently, with the advent of precision measurements of the anisotropy of the cosmic microwave background on very small angular scales [27], the standard theory has actually run into a potential problem, with it proving to be somewhat difficult [28] to fit the anisotropy data (and particularly their apparent lack of any second

¹³Even though the cosmology expands far more slowly than the standard one, nonetheless, this gets compensated for in the conformal case because weak interactions are then found [23] to remain in thermal equilibrium down to lower temperatures than in the standard case, with the conformal metallicity predictions then being found [23] to actually outperform those of the standard model.

¹⁴Even though spallation models of deuterium production were never actually ruled out, the models were quickly set aside once it became apparent that deuterium could be produced by standard big bang nucleosynthesis.

¹⁵It could thus be of interest to measure the lithium to deuterium abundance ratio of high z quasar absorbers, with the obtaining of a value for this ratio different from that expected in standard big bang nucleosynthesis then possibly indicating the occurrence of inhomogeneous but still fairly early universe spallation.

acoustic peak) using the baryon density which is explicitly inferred from big bang nucleosynthesis.¹⁶ And while it is of course far too early to draw any permanent conclusions, nonetheless the current situation could be another indicator, albeit so far only a mild one, that G might not in fact control cosmology.

Also, of course, by the very same token, these selfsame anisotropy data also provide a new challenge to alternate gravitational theories as well, with it therefore being somewhat urgent to determine the specific anisotropy predictions of the conformal theory. And with conformal gravity being confined to the rather tight $\bar{\Omega}_M(t_0) = 0$, $0 \leq \bar{\Omega}_\Lambda(t_0) \leq 1$, $1 \geq \Omega_k(t_0) \geq 0$ window (a window which obliges $\bar{\Omega}_M(t)$ to still be negligible at recombination), such predictions may well prove to be definitive for the theory.¹⁷ Thus again it becomes necessary to develop a theory for the growth of conformal cosmology inhomogeneities. (Essentially, with the global cosmic background being homogeneous, and with local gravity only arising through inhomogeneities in the conformal gravity theory, galaxy fluctuation theory sits just at the point where local and global conformal effects become competitive.)

While conformal gravity can thus be definitively tested at recombination, it is important to note that there is actually a much nearer by redshift at which definitive testing can also be made, namely [29] that provided by only a modest extension of the Hubble plot a little beyond $z = 1$. Specifically, we had noted earlier that the standard model actually predicts that the universe be decelerating above $z = 1$ (a thus definitive test for it in and of itself) while the conformal gravity theory continues to be accelerating above $z = 1$ ($q(t, \alpha > 0, k < 0$) of Eq. (23) is always negative); with Hubble plot data at around $z = 2$ (where the conformal theory is dimmer than the standard theory by about 0.3 magnitudes) thus enabling one to make a fairly definitive discrimination between the two theories. Specifically, in [29] it was shown that the luminosity distance redshift relation associated with $\alpha > 0$, $k < 0$ conformal gravity is given as

$$H(t_0)d_L/c = -(1+z)^2 \left\{ 1 - [1 + q(t_0) - q(t_0)/(1+z)^2]^{1/2} \right\} / q(t_0), \quad (29)$$

with the ensuing conformal gravity fits to the $z < 1$ data presented in [29] being found to be every bit as good as the corresponding $\Omega_M(t_0) = 0.3$, $\Omega_\Lambda(t_0) = 0.7$ standard model fits. Specifically, for 54 fitted $z < 1$ data points of [2] and [30], $q(t_0) = -0.37$ conformal gravity gives $\chi^2 = 58.6$, $q(t_0) = 0$ conformal gravity gives $\chi^2 = 61.5$, while $\Omega_M(t_0) = 0.3$,

¹⁶While the standard theory had always been able to fit the light element abundances with a very specific phenomenological value for the baryon density (as determined solely from the abundance fitting itself), we note in passing that there had never actually been any independent confirmation of that particular value.

¹⁷These numbers should not be compared with the currently preferred standard model $\Omega_M(t_0) \approx 0.3$, $\Omega_\Lambda(t_0) \approx 0.7$, $\Omega_k(t_0) \approx 0$ numbers (see e.g. [27]), since the extraction of those numbers from data requires the assumption of a dynamical early universe fluctuation model (as well as the use, of course, of a completely mysterious and not understood value for $\Omega_\Lambda(t_0)$). It is for that reason that we have focused on the supernovae data in the present paper, since we can extract information from them (such as a sign and magnitude for $q(t_0)$) without recourse to any particular dynamical model at all.

$\Omega_\Lambda(t_0) = 0.7$ standard gravity gives $\chi^2 = 57.7$, with the predictions of the two theories thus being completely indistinguishable below $z < 1$, and with one therefore needing to go above $z = 1$ to discriminate. Thus despite the longstanding conviction that ordinary matter is a substantial contributor to cosmology in the nearby universe, available $z < 1$ data are just as compatible with ordinary matter making no contribution at all (viz. $\bar{\Omega}_M(t_0) = 0$).

Now it would be quite remarkable if two totally different theories could account for the very same $z < 1$ data, and it is thus worthwhile to identify why this occurs. To this end we note that all $\Omega_k(t_0) = 0$ standard model fits have to lie between its ($\Omega_M(t_0) = 1, \Omega_\Lambda(t_0) = 0$) and ($\Omega_M(t_0) = 0, \Omega_\Lambda(t_0) = 1$) limits, limits for which the respective luminosity functions are given by the (too bright for the data) $d_L = cH(t_0)^{-1}2(1+z)[1 - (1+z)^{-1/2}]$ and the (too dim) $d_L = cH(t_0)^{-1}(z+z^2)$, and for which the respective χ^2 for the 54 $z < 1$ data points are found to be $\chi^2 = 92.9$ and $\chi^2 = 75.8$. Now while both of these χ^2 values are sufficiently far from the data so as to exclude them, they are not overwhelmingly far from the data (since z does not get large enough to cause the various expectations for d_L to differ that much), thus making it possible for some intermediate (not too bright, not too dim) prediction such as ($\Omega_M(t_0) = 0.3, \Omega_\Lambda(t_0) = 0.7$) to then work. Similarly, the $\Omega_k(t_0) = 1$ (viz. $q(t_0) = 0$) conformal gravity prediction (the not too bright, not too dim $d_L = cH(t_0)^{-1}(z+z^2/2)$) also does not differ from them that much either at $z < 1$ allowing it to work well too. The data below $z = 1$ can thus be accounted for by the universe being near to either $\Omega_k(t_0) = 0$ or to $\Omega_k(t_0) = 1$, values for $\Omega_k(t_0)$ which are not orders of magnitude apart, with it being necessary to go to higher z to discriminate. Thus it could be that the standard model is correct and that the $\Omega_k(t_0) = 1$ conformal gravity fit is fortuitous, or it could be that conformal gravity is correct with it then being the $\Omega_k(t_0) = 0$ standard model fit which is the fortuitous one, with this issue then readily being resolved as soon as the Hubble plot can be definitively pushed beyond $z = 1$.

Also conformal gravity can of course equally be tested at altogether higher redshifts such as at those associated with the cosmic microwave background and nucleosynthesis, and while one should not understate the seriousness of the inhomogeneity and deuterium problems for conformal cosmology (indeed its very viability as a cosmological model is contingent upon a successful resolution of these very issues), nonetheless, the relative ease with which conformal gravity deals with cosmological constant problem, the most severe problem the standard theory faces, would appear to entitle the conformal theory to further consideration. And even if the conformal gravity alternative were to fall by the wayside, nonetheless our analysis of the role that G plays in engendering the standard model cosmological constant problem would still remain valid. Moreover, the cosmological constant problem is not the only problem which besets standard gravity, with dark matter theory itself having steadily accumulated a whole host of unresolved challenges of its own [31], especially in the area of galactic dynamics. And indeed, after identifying more than 10 distinct such problems, the authors of [31] venture to suggest that it is no longer obvious that the formidable difficulties facing alternate theories are any more daunting than those facing dark matter.

To conclude this paper, we note once again that spontaneous breakdown effects such as those associated with a Goldstone boson pion or with massive intermediate vector bosons seem to be very much in evidence in current era particle physics experiments, and are thus not quenched at all apparently. Hence all the evidence of particle physics is that its contribution to Λ should in fact be large rather than small today. However, since in such

a case it is nonetheless possible for $\bar{\Omega}_\Lambda(t_0)$ to still be small today, we see that the standard gravity fine tuning problem associated with having $\Omega_M(t_0) \simeq \Omega_\Lambda(t_0)$ today can be viewed as being not so much one of trying to understand why it is $\Omega_\Lambda(t_0)$ which is of order one after 15 or so billion years, but rather of trying to explain why the matter density contribution to cosmology should be of order one after that much time rather than a factor T^4/T_V^4 smaller. Since this latter problem is readily resolved if G does not in fact control cosmology, but if cosmology is instead controlled by some altogether smaller length squared scale such as $G_{eff} = -3c^3/4\pi\hbar S_0^2$, we see that the origin of the entire cosmological constant problem can be directly traced to the assumption that gravity is controlled by Newton's constant G on each and every distance scale; with the very existence of the cosmological constant problem possibly being an indicator that the extrapolation of standard gravity from its solar system origins all the way to cosmology might be a lot less reliable than is commonly believed.

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REFERENCES

- [1] A. G. Riess et. al., *Astronom. J.* **116**, 1009 (1998).
- [2] S. Perlmutter et. al., *Astrophys. J.* **517**, 565 (1999).
- [3] A. Dev, M. Sethi and D. Lohiya, "Linear Coasting in Cosmology and SNe 1a", *astro-ph/0008193* (2000).
- [4] S. Weinberg, *Rev. Mod. Phys.* **61**, 1 (1989).
- [5] Y. J. Ng, *Int. J. Mod. Phys. D* **1**, 145 (1992).
- [6] A. H. Guth, *Phys. Rev. D* **23**, 347 (1981).
- [7] P. D. Mannheim, *Astrophys. J.* **391**, 429 (1992).
- [8] P. D. Mannheim, *Phys. Rev. D* **58**, 103511 (1998).
- [9] H. Weyl, *Math. Zeit.*, **2**, 384 (1918).
- [10] S. L. Adler, *Rev. Mod. Phys.* **54**, 729 (1982).
- [11] A. Zee, *Ann. Phys.* **151**, 431 (1983).
- [12] P. D. Mannheim and D. Kazanas, *Gen. Relativ. Gravit.* **26**, 337 (1994).
- [13] A. S. Eddington, *The Mathematical Theory of Relativity*, Eighth Edition (First Edition 1922), Cambridge University Press, Cambridge, U. K. (1960).
- [14] B. S. DeWitt, *Dynamical Theory of Groups and Fields*, Gordon and Breach, NY (1965).
- [15] P. D. Mannheim and D. Kazanas, *Astrophys. J.* **342**, 635 (1989).
- [16] P. D. Mannheim, *Astrophys. J.* **479**, 659 (1997).
- [17] P. D. Mannheim, *Gen. Relativ. Gravit.* **22**, 289 (1990).
- [18] P. D. Mannheim, *Conformal Cosmology and the Age of the Universe*, *astro-ph/9601071* (1996).
- [19] P. D. Mannheim, *Found. Phys.* **30**, 709 (2000).
- [20] S. S. McGaugh, in *Proceedings of Galaxy Dynamics*, A Rutgers Symposium, August 1998. *Astronomical Society of the Pacific Conference Series*, Vol. **182**, edited by D. Merritt, J. A. Sellwood, and M. Valluri, A. S. P., San Francisco (1999).
- [21] L. Knox and A. Kosowsky, "Primordial Nucleosynthesis in Conformal Weyl Gravity", *Fermilab-Pub-93/322-A*, *astro-ph/9311006* (1993).
- [22] D. Elizondo and G. Yepes, *Astrophys. J.* **428**, 17 (1994).
- [23] D. Lohiya, A. Batra, S. Mahajan, and A. Mukherjee, "Nucleosynthesis in a Simmering Universe", *nucl-th/9902022* (1999); M. Sethi, A. Batra, and D. Lohiya, *Phys. Rev. D* **60**, 108301 (1999).
- [24] F. Hoyle and W. A. Fowler, *Nature* **241**, 384 (1973).
- [25] R. I. Epstein, J. M. Lattimer, and D. N. Schramm, *Nature* **263**, 198 (1976).
- [26] R. I. Epstein, *Astrophys. J.* **212**, 595 (1977).
- [27] P. de Bernardis et. al., *Nature* **404**, 955 (2000).
- [28] M. Tegmark and M. Zaldarriaga, *Phys. Rev. Lett.* **85**, 2240 (2000).
- [29] P. D. Mannheim, *How Recent is Cosmic Acceleration?*, *astro-ph/0104022* (2001).
- [30] M. Hamuy et. al., *Astronom. J.* **112**, 2391 (1996).
- [31] J. A. Sellwood and A. Kosowsky, "Does Dark Matter Exist?", *astro-ph/0009074* (2000).